Comparing Quantum Black Holes and Naked Singularities[1]

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Abstract

There are models of gravitational collapse in classical general relativity which admit the formation of naked singularities as well as black holes. These include fluid models as well as models with scalar fields as matter. Even if fluid models were to be regarded as unphysical in their matter content, the remaining class of models (based on scalar fields) generically admit the formation of visible regions of finite but arbitrarily high curvature. Hence it is of interest to ask, from the point of view of astrophysics, as to what a stellar collapse leading to a naked singularity (or to a visible region of very high curvature) will look like, to a far away observer. The emission of energy during such a process may be divided into three phases - (i) the classical phase, during which matter and gravity can both be treated according to the laws of classical physics, (ii) the semiclassical phase, when gravity is treated classically but matter behaves as a quantum field, and (iii) the quantum gravitational phase. In this review, we first give a summary of the status of naked singularities in classical relativity, and then report some recent results comparing the semiclassical phase of black holes with the semiclassical phase of spherical collapse leading to a naked singularity. In particular, we ask how the quantum particle creation during the collapse leading to a naked singularity compares with the Hawking radiation from a star collapsing to form a black hole. It turns out that there is a fundamental difference between the two cases. A spherical naked star emits only about one Planck energy during its semiclassical phase, and the further evolution can only be determined by the laws of quantum gravity. This contrasts with the semiclassical evaporation of a black hole wherein gravity can be treated classically all the way till the final stages of evaporation until a Planck mass remnant remains. Hence spherical collapse leading to a naked singularity provides an interesting and promising system for testing our understanding of quantum gravity. The results on the semiclassical phase of naked collapse reviewed here have been obtained in collaboration with Barve, Harada, Iguchi, Nakao, Tanaka, Vaz and Witten

1   Cosmic Censorship and Classical General Relativity

Put in broad physical terms, the Cosmic Censorship Hypothesis states that *generic* gravitational collapse of *physically reasonable* matter does not end in the formation of a *naked singularity*. A naked singularity is a singularity which is visible to a far away observer, i.e. outgoing light rays starting from the singularity terminate on the singularity in the past. By 'physically reasonable' we mean matter which satisfies one or more positive energy conditions - matter which can in principle be prepared in the laboratory. By 'generic' we mean that the initial conditions leading to the formation of the naked singularity are not special (of zero measure).

The Censorship Hypothesis is important in classical general relativity because there are theorems, for instance the black hole area theorem, whose proof assumes the validity of the Hypothesis. However, it is not obvious a priori that the results of these theorems cannot be proved without assuming Censorship. This by itself is an interesting direction for research in the classical theory.

The issue of naked singularities is not trivialized by quantum gravity, even though it might be true that quantum gravity will remove the singularities of classical general relativity, by replacing them by a smeared out region of Planck scale curvature. If the classical singularity resides inside an astrophysical black hole,

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[1]Based on a talk given at JGRG10, Osaka, September, 2000. To appear in the conference proceedings.

such a smeared region will be invisible to a distant observer. However, if the classical singularity is naked, the Planck curvature region by which it will be replaced will be visible to the distant observer and will have physical consequences very different from that of the black hole. For instance, it could give rise to an intense particle production which could differ in character from the Hawking radiation from a black hole.

If we think of classical general relativity as a limit of quantum gravity, then we should think of Cosmic Censorship in a broader perspective, and take the Hypothesis to mean: 'Effects of quantum gravity cannot be observed in gravitational collapse'. Looked at in this way, it would be more useful for physics if Censorship were to be violated, as then the Universe could consist of naked stars - stars which according to the classical theory end as naked singularities, and in whose collapse we may witness the effects of quantum gravity.

Over the last twenty years or so, there have been many investigations of Cosmic Censorship, largely using spherical models of gravitational collapse of physically reasonable matter. The simplest of these models is dust collapse, which has been studied in great detail and it has been shown that both black holes and naked singularities form from generic initial conditions. Eardley and Smarr [1] used numerical methods to study the dust model. Christodoulou [2] analysed dust collapse with C^∞ initial data and Newman [3] showed that the consequent naked singularities were gravitationally weak. Dwivedi and Joshi developed a roots analysis to check naked singularity formation [4] and demonstrated the occurrence of strong curvature naked singularities in dust collapse. Waugh and Lake [6] showed the occurrence of strong curvature naked singularity in self-similar dust collapse. The initial data leading to these strong curvature naked singularities was worked out in [7] and a unified derivation of strong and weak naked singularities was obtained in [8]. The structure of the apparent horizon in dust collapse was studied in [9] and a simplified derivation of the dust naked singularity was given in [10]. Harada et al. [11, 12] investigated the stability of spherical dust collapse against non-spherical perturbations. The strength of the singularity has been studied by [13], the nature of non-spacelike geodesics in [14] and the role of initial data in [15, 16]. A cosmological constant has been included in [17, 18].

Similar results have been obtained for null dust collapse described by the Vaidya model [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

The next class of models is spherical perfect fluid collapse, which has been examined numerically [31] and analytically [32, 33] with and without the assumption of self-similarity [34, 35] and again it has been found that both black hole and naked singular solutions arise from generic initial data.

Another tractable class of models is the spherical collapse of fluids which have only tangential pressure, and for which the radial pressure is assumed to be negligible. For these models also it is observed that covered as well as naked solutions exist [36, 37, 38, 39, 40, 41, 42].

One might consider dividing physically reasonable matter models into two classes - fluids and fields. The former may be regarded as a phenomenological description of matter while the latter may be regarded as fundamental. Could it be that naked singularities do not arise generically in the collapse of matter which is treated as a field? The work of Christodoulou [43, 44, 45, 46, 47, 48, 49] on massless scalar fields suggests this could be the case. It should be said though that this issue of phenomenological versus fundamental needs further investigation, and it is not entirely clear why fluids should admit naked singularities but fields should not.

Even if one were to accept that Cosmic Censorship should be investigated with only fundamental matter, there is a broader outlook which in my view seals the case against Censorship. This has to do with the result that in spherical scalar collapse one can form black holes of arbitrarily small mass [50]. Hence, although the naked singularity itself forms from non-generic initial data, visible regions of arbitrarily high curvature (the curvature on the horizon of the arbitrarily small black hole) form generically.

So it may turn out to be true that the collapse of fundamental matter does not generically form naked singularities. Yet it appears to be more physical to think of Cosmic Censorship as a statement not about singularities, but about regions of extremely high curvature (approaching Planck scale) and in this sense censorship does not appear to hold in spherical general relativity, even if only fundamental matter is considered.

Attempts have also been made to study the nature of singularities in spherical collapse without having to restrict to a specific form of matter [51, 52, 53, 54, 55]

If we accept that Censorship does not hold, in the sense described in the previous paragraph, are there any interesting implications for astrophysics? This is far from clear. We have to address the following questions: if a star collapses to form a naked singularity (or a black hole of very small mass) what does the process look like to a far off observer? How much energy is emitted and over what duration and with what spectrum? How does the back reaction from the emission modify the collapse? In this paper, we address some of these questions.

Let us broadly divide 'censorship violating' spherical collapse into two classes: one which results in a naked singularity at the center, and the second which results in a black hole of very small mass while the rest of the mass disperses to infinity. In this article we focus attention on the first of these two classes.

The emission from a star collapsing to form a naked singularity may be divided into three phases - classical, semiclassical and quantum gravitational. The classical phase would consist of the radiation which is emitted while both matter and gravity can be treated by the laws of classical physics. Could most of the star blow itself up during this phase? We do not know for sure. This phase has been investigated by Joshi, Dadhich and Maartens [56]. In the next phase of the collapse, matter will have to be treated as a quantum field whereas gravity can be treated classically (the semiclassical phase). This is the analog of the black hole phase in which the black hole emits Hawking radiation. The final collapse phase is quantum gravitational.

The main purpose of the present article is to report some recent results of Harada et al. [57] on the semiclassical phase of a spherical star forming a naked singularity. We show that the nature of particle creation in this phase is very different from the Hawking radiation from a black hole and that quantum gravity is essential for understanding the gravitational collapse to a naked singularity.

Some of the earlier reviews on Cosmic Censorship are listed in [58, 59, 60, 61, 62, 63, 64, 65]. The reader might find it useful to read the present article in conjunction with some of the earlier reviews.

2 Quantum Black Holes Versus Quantum Naked Singularities

Consider a spherical star which collapses to form a black hole and another spherical star which collapses to form a naked singularity. Let us quantize a massless scalar field on the background of either star, and prepare the field to be initially in the Minkowski vacuum, when the gravitational field of the star is weak enough so that spacetime is nearly flat. As is well known, the star which settles into the black hole state emits Hawking radiation, which has a thermal spectrum with temperature inversely proportional to the mass of the black hole.

We are interested in finding out the nature of the corresponding quantum radiation emitted in the naked singularity case. A comparison of the representative Penrose diagrams (Fig. 1) in the two cases
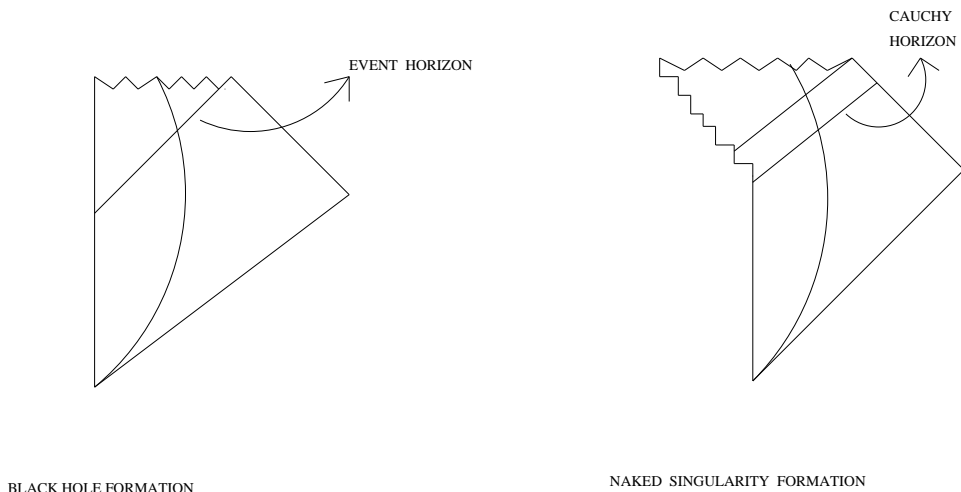

FIGURE 1.

shows the immediate obstacle to repeating Hawking's Bogoliubov transformation calculation in the naked case. In the latter case, a part of the asymptotic future null infinity is exposed to the naked null singularity, making it impossible to define a complete set of modes on $\mathcal{I}^+$ unless some boundary conditions are prescribed on the naked singularity. Hence it is not possible to carry out the Bogoliubov transformations and a particle creation calculation the way it is done for the black hole case.

However, there is in principle no problem in calculating the expectation value of the quantum stress energy tensor of the scalar field in the region of the Penrose diagram prior to the Cauchy horizon. From this, one can obtain the expectation value of the outgoing quantum flux everywhere in that part of spacetime which is not exposed to the naked singularity, and in particular on that part of $\mathcal{I}^+$ which is prior to its intersection with the Cauchy horizon. Understandably, for the black hole case, such a calculation yields the standard Hawking thermal flux. We are now asking for the result of this calculation in the case when the star collapses to a naked singularity.

In the case of a four-dimensional spherical collapse Ford and Parker [66] obtained a formula for the outgoing quantum flux on the Cauchy horizon in the geometric optics approximation, using point-splitting regularisation. Suppose an ingoing ray $V = constant$ coming in from $\mathcal{I}^-$ gets mapped into an outgoing ray $U = \mathcal{F}(V)$ when it reaches $\mathcal{I}^+$. Then the outgoing quantum flux, for a given low angular momentum mode, is

$$P = \frac{\hbar}{24\pi} \left[\frac{\mathcal{F}'''}{\mathcal{F}'^3} - \frac{3}{2} \left(\frac{\mathcal{F}''}{\mathcal{F}'^2} \right)^2 \right]. \tag{1}$$

In terms of the inverse map $V = \mathcal{G}(U)$ the power is given by

$$P = \frac{\hbar}{24\pi} \left[\frac{3}{2} \left(\frac{\mathcal{G}''}{\mathcal{G}'^2} \right)^2 - \frac{\mathcal{G}'''}{\mathcal{G}'} \right]. \tag{2}$$

For a two-dimensional spacetime model, the outgoing quantum flux can be calculated exactly. Suppose the spacetime is given by double null coordinates u, v as

$$ds^2 = C^2(u, v) du dv. \tag{3}$$

Then, the use of the trace anomaly and the divergencelessness condition

$$Tr < in|T_{\mu\nu}|in > = \frac{\mathcal{R}}{24\pi}, \quad < T^{\mu\nu} >_{;\nu} = 0 \tag{4}$$

allows the outgoing flux to be written as [67]

$$< T_{uu} > = -\frac{1}{12\pi} C \frac{\partial^2}{\partial u^2} (1/C). \tag{5}$$

Here, $\mathcal{R}$ is the curvature scalar for the two dimensional spacetime. In the four dimensional case as well as for the 2-d approximation, one needs to be able to solve for null rays in order to calculate the flux. We now discuss these two approximations one by one. Before doing that, we give a brief outline of the naked singularity in the classical spherical dust collapse model [10].

Spherical Dust Collapse

In comoving coordinates (t, r, θ, ϕ) the spacetime metric for marginally bound spherical dust collapse is given by

$$ds^2 = dt^2 - R'^2 dr^2 - R^2 d\Omega^2 \tag{6}$$

where $R(t, r)$ is the area radius at time t of the shell having the comoving coordinate r. A prime denotes partial derivative w.r.t. r. The energy-momentum tensor for dust has only one non-zero component $T_0^0 = \epsilon(t, r)$, which is the energy density. The Einstein equations for the collapsing cloud are

$$\frac{8\pi G}{c^4} \epsilon(t, r) = \frac{F'}{R^2 R'}, \quad \dot{R}^2 = \frac{F(r)}{R}. \tag{7}$$

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A dot denotes partial derivative w.r.t. time t. The function $F(r)$ results from the integration of the second order equations. Henceforth we shall set $8\pi G/c^4 = 1$.

The second of these equations can be easily solved to get

$$R^{3/2}(t, r) = r^{3/2} - \frac{3}{2}\sqrt{F}t \tag{8}$$

where we have used the freedom in the scaling of the comoving coordinate r to set $R(0, r) = r$ at the starting epoch of collapse, $t = 0$. It follows from the first equation in (7) that the function $F(r)$ gets fixed once the initial density distribution $\epsilon(0, r) = \rho(r)$ is given, i.e.

$$F(r) = \int \rho(r)r^2 \, dr. \tag{9}$$

Hence $F(r)$ has the interpretation of being twice the mass to the interior of the shell labeled r. If the initial density $\rho(r)$ has a series expansion

$$\rho(r) = \rho_0 + \rho_1 r + \frac{1}{2!}\rho_2 r^2 + \frac{1}{3!}\rho_3 r^3 + \dots \tag{10}$$

near the center $r = 0$, the resulting series expansion for the mass function $F(r)$ is

$$F(r) = F_0 r^3 + F_1 r^4 + F_2 r^5 + F_3 r^6 + \dots \tag{11}$$

where $F_q = \rho_q/q!(q+3)$, and $q = 0, 1, 2, 3..$We note that we could set $\rho_1 = 0$ without in any way affecting the conclusions of this paper. Further, the first non-vanishing derivative in the series expansion in (10) should be negative, as we will consider only density functions which decrease as one moves out from the center.

According to (8) the area radius of the shell r shrinks to zero at the time $t_c(r)$ given by

$$t_c(r) = \frac{2r^{3/2}}{3\sqrt{F(r)}}. \tag{12}$$

At $t = t_c(r)$ the Kretschmann scalar

$$K = 12\frac{F'^2}{R^4 R'^2} - 32\frac{FF'}{R^5 R'} + 48\frac{F^2}{R^6} \tag{13}$$

diverges at the shell labeled r and hence this represents the formation of a curvature singularity at r. In particular, the central singularity, i.e. the one at $r = 0$, forms at the time

$$t_0 = \frac{2}{3\sqrt{F_0}} = \frac{2}{\sqrt{3\rho_0}}. \tag{14}$$

At $t = t_0$ the Kretschmann scalar diverges at $r = 0$. Near $r = 0$, we can expand $F(r)$ and approximately write for the singularity curve

$$t_c(r) = t_0 - \frac{F_n}{3F_0^{3/2}}r^n. \tag{15}$$

Here, F_n is the first non-vanishing term beyond F_0 in the expansion (11). We note that $t_c(r) > t_0$, since F_n is negative.

We wish to investigate if the singularity at $t = t_0, r = 0$ is naked, i.e. are there one or more outgoing null geodesics which terminate in the past at the central singularity. We restrict attention to radial null geodesics. Let us start by assuming that one or more such geodesics exist, and then checking if this assumption is correct. Let us take the geodesic to have the form

$$t = t_0 + ar^\alpha \tag{16}$$

to leading order, in the $t - r$ plane, where $a > 0$, $\alpha > 0$. In order for this geodesic to lie in the spacetime, we conclude by comparing with (15) that $\alpha \geq n$, and in addition, if $\alpha = n$, then $a < -F_n/3F_0^{3/2}$.

As is evident from the form (6) of the metric, an outgoing null geodesic must satisfy the equation

$$\frac{dt}{dr} = R'. \tag{17}$$

In order to calculate R' near $r = 0$ we first write the solution (8) with only the leading term F_n retained in $F(r)$ in (11). This gives

$$R = r \left(1 - \frac{3}{2}\sqrt{F_0}\left[1 + \frac{F_n}{2F_0}r^n\right]t\right)^{2/3}. \tag{18}$$

Differentiating this w.r.t. r gives

$$R' = \left(1 - \frac{3}{2}\sqrt{F_0}\left[1 + \frac{F_n}{2F_0}r^n\right]t\right)^{-1/3}\left(1 - \frac{3}{2}\sqrt{F_0}t - \frac{(2n+3)F_n}{4\sqrt{F_0}}r^n t\right). \tag{19}$$

Along the assumed geodesic, t is given by (16). Substituting this in R' and equating the resulting R' to $dt/dr = \alpha a r^{\alpha-1}$ gives

$$\alpha a r^{\alpha-1} = \frac{\left(1 - \frac{3}{2}\sqrt{F_0}\left[t_0 + ar^\alpha\right] - \frac{(2n+3)F_n}{4\sqrt{F_0}}r^n\left[t_0 + ar^\alpha\right]\right)}{\left(1 - \frac{3}{2}\sqrt{F_0}\left[1 + \frac{F_n}{2F_0}r^n\right]\left[t_0 + ar^\alpha\right]\right)^{1/3}}. \tag{20}$$

This is the key equation. If it admits a self-consistent solution then the singularity will be naked (i.e. at least one outgoing null geodesic will terminate at the singularity), otherwise not. We simplify this equation by putting in the requirement mentioned earlier, that $\alpha \geq n$. Consider first $\alpha > n$. In this case we get, to leading order

$$\alpha a r^{\alpha-1} = \left(1 + \frac{2n}{3}\right)\left(-\frac{F_n}{2F_0}\right)^{2/3}r^{2n/3} \tag{21}$$

which implies that $\alpha = 1 + 2n/3$, and $a = (-F_n/2F_0)^{2/3}$. By substituting integral values for n we find that only for $n = 1$ and $n = 2$ the condition $\alpha > n$ is satisfied. Hence the singularity is naked for $n = 1$ and $n = 2$, i.e. for the models $\rho_1 < 0$ and for $\rho_1 = 0, \rho_2 < 0$. There is at least one outgoing geodesic given by (16) which terminates in the central singularity in the past. If $n > 3$ then the condition $\alpha > n$ cannot be satisfied and the singularity is not naked. This is the case $\rho_1 = \rho_2 = \rho_3 = 0$. It includes as a special case the homogeneous black hole model of Oppenheimer and Snyder.

Consider next that $\alpha = n$. In this case we get from (20) that

$$nar^{n-1} = \frac{-\frac{3}{2}a\sqrt{F_0} - \frac{(2n+3)F_n}{6\sqrt{F_0}}}{\left(-\frac{F_n}{2F_0} - \frac{3a}{2}\sqrt{F_0}\right)^{1/3}}r^{2n/3} \tag{22}$$

which implies that $n = 3$ and gives an implicit expression for a in terms of F_3 and F_0. This expression for a can be simplified to get the following quartic for a:

$$12\sqrt{F_0}a^4 - a^3(-4F_3/F_0 + F_0^{3/2}) - 3F_3a^2 - 3F_3^2/F_0^{3/2}a - (F_3/F_0)^3 = 0. \tag{23}$$

By defining $b = a/F_0$ and $\xi = F_3/F_0^{5/2}$ this quartic can be written as

$$4b^3(3b + \xi) - (b + \xi)^3 = 0. \tag{24}$$

The singularity will be naked if this equation admits one or more positive roots for b which satisfy the constraint $b < -\xi/3$. This last inequality is the same as the condition $a < -F_n/3F_0^{3/2}$ given below equation (16). We note that ξ is negative. This quartic can be made amenable to further analysis by substituting $Y = -2b/\xi$, and then $\eta = -1/6\xi$, so as to get

$$Y^3(Y - 2/3) - \eta(Y - 2)^3 = 0. \tag{25}$$

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This quartic has two positive real roots provided $\eta \geq \eta_1$ or $\eta \leq \eta_2$ where

$$\eta_1 = \frac{26}{3} + 5\sqrt{3}, \ \eta_2 = \frac{26}{3} - 5\sqrt{3}. \tag{26}$$

We also require that $Y < 2/3$. By examining the quartic (25) one can see that if $\eta \geq \eta_1$ then $Y \geq 2$; hence this range of η is ruled out. Thus the singularity is naked provided $\eta \leq \eta_2$, or equivalently $\xi \leq -25.9904$.

The above results give the conditions on the initial data for which the singularity is at least locally naked, i.e. light manages to escape from the singularity. Further constraints have to be put on the initial distribution in order to obtain a globally naked singularity, i.e. one in which light rays starting from the singularity escape the dust cloud entirely. In general a choice of initial conditions is available for which a singularity which is at least locally naked is also globally naked. The spacetime with a globally naked singularity is described by the Penrose diagram in Figure 1. From the point of view of the present article the globally naked singularities are the interesting ones; they are the ones whose asymptotic quantum appearance differs from those of black holes.

A useful special case of the class $\alpha = n = 3$ is the self-similar model, one which admits a homothetic Killing vector field [31]. The condition for nakedness is the same as for the whole class $\alpha = n = 3$ and the singularity is necessarily globally naked.

We recall next the quantisation of a massless scalar field on the background spacetime provided by dust collapse.

Geometric Optics Approximation

We now consider the application of the Ford-Parker formula (2) for calculating the quantum flux of a massless scalar field on the dust background. For the case in which the collapse ends in a black hole, the map $\mathcal{G}(U)$ is given by

$$\mathcal{G}(U) = A - B\exp(-U/4M). \tag{27}$$

Using this map in (2) gives the expected thermal Hawking flux

$$P = \frac{1}{768\pi M^2}. \tag{28}$$

The Hawking flux for the black hole is found using the map $\mathcal{F}(v)$ which in turn depends on the behaviour of null rays in the vicinity of the event horizon. In this sense the map is universal for black holes - it does not depend on the interior geometry of the star.

On the other hand the map $\mathcal{G}(U)$ very much depends on the internal geometry in the naked case. Hence one has to first specify the internal solution and calculate the propagation of null rays inside the star. There are only a limited number of cases where this has been done so far. The first example to be considered was that of shell-crossing dust naked singularity [66] for which the outgoing flux was calculated and shown to be finite.

Of greater interest is the shell-focusing naked central singularity in spherical dust collapse, described above. This was considered by Barve et al. [68] for the self-similar dust collapse and by Singh and Vaz [69] for self-similar null dust collapse. The map $\mathcal{G}(U)$ was calculated and shown to be

$$\mathcal{G}(U) = A - B(V_0 - V)^\gamma \tag{29}$$

where γ is a positive constant determined by the collapse model. The flux can be calculated using (2) and near the Cauchy horizon $U = U_0$ it behaves as

$$P(U) \sim \frac{\hbar}{(U_0 - U)^2}. \tag{30}$$

It thus diverges in the approach to the Cauchy horizon. Such a divergence appears to be a universal feature of collapse leading to a naked singularity. It has also been observed in dust collapse models which do not assume self-similarity [70, 71], for which the divergence is of the form

$$P(U) \sim \frac{\hbar}{(U_0 - U)^{3/2}}. \tag{31}$$

What is the meaning of this divergence? At first sight, it suggests that as a result of the intense particle creation accompanying the formation of the naked singularity, a huge amount of energy is emitted - essentially the whole star blows up. However, we realize that in inferring this divergence we have extended the semiclassical approximation all the way up to the epoch of naked singularity formation, when the curvature of the background spacetime diverges. But we should not a priori trust the results of the semiclassical theory beyond the epoch when the curvature in the central region of the star approaches Planck scale. This happens about one Planck time before the singular epoch, and this can be shown [57] to translate to $U_0 - U \approx t_{Planck}$ in (30). It then follows from (30) that during the domain of validity of the semiclassical approximation the emitted energy is only about one Planck energy, even if the total initial mass of the star is much larger, say a solar mass. Hence this system is very different from the black hole, for which essentially the entire mass of the collapsing object evaporates during the semiclassical phase and Planck scale curvatures are realised only when a Planck mass remnant remains.

The result that the semiclassical energy emission is very small appears to be supported by the observation that the map $\mathcal{F}(V)$ in the naked case is determined by examining a small spacetime region near the central singularity [72].

We mentioned earlier that the calculation of the spectrum of created particles, which is based on Bogoliubov transformations, cannot be performed in the naked case without specifying boundary conditions on the singularity. If the semiclassical approximation were assumed to be valid all the way up to the formation of the naked singularity, the resulting divergence on the Cauchy horizon would suggest a complete evaporation of the star. Under such a situation it appears reasonable to assume that the singularity does not form in the first place, and an appropriate boundary condition would be that of no emission from the null singularity. Under these assumptions it has been shown that the emitted spectrum is non-thermal [73, 74, 71]. It however remains to be understood how these results on the spectrum are modified by the quantum gravity phase which comes into play about a Planck time prior to the singularity.

2-d collapse models

The 2-d models that we consider are obtained by suppressing the angular coordinates in a 4-d spherical model. If one applies the 2-d quantum flux result to the Schwarzschild geometry, one again obtains the Hawking result (28) for the power. The first application to a naked singularity model was by Hiscock et al. [19] who calculated the outgoing flux for the naked singularity in self-similar Vaidya null dust collapse and showed it to diverge on the Cauchy horizon. A similar divergence was found by Barve et al. [75] for the case of self-similar Tolman-Bondi dust collapse:

$$< T_{uu} > \sim \frac{\hbar}{(z - z_-)^2} \tag{32}$$

where z is the self-similarity parameter and $z = z_-$ is the Cauchy horizon. It was shown in [76] that the divergence on the Cauchy horizon persists in non-self-similar 2-d dust collapse.

Once again, it is apparent that the semiclassical evolution should be stopped about a Planck time before the singular epoch. It then turns out that in the 2-d model as well the energy emitted during the semiclassical phase is about one Planck energy.

An important aspect of the above analysis is that we have ignored the back-reaction, and one could well ask if our conclusions could be affected by its inclusion. We now give arguments as to why the back reaction cannot be important during the semiclassical evolution, in the present case. One could propose two different criteria for deciding as to when the back reaction becomes important. One is that the total flux received at infinity becomes comparable to the mass of the collapsing star. As we have seen above, if the mass of the collapsing star is much greater than the Planck mass (as is of course usually the case) then the back reaction does not become important during the semiclassical phase.

The second criterion could be that the energy density of the quantum field becomes comparable to the background energy density, *inside* the star. In a four-dimensional model, it is difficult to establish whether this happens. However, one can study the evolution of the quantum field inside the star using the 2-d model obtained by suppressing the angular part of the 4-d Tolman-Bondi model [75]. Using this model, Iguchi and Harada [77] have shown that the back reaction does not become significant during the semiclassical evolution, inside the star. It is plausible that this result holds for the four dimensional

stellar dust model as well, in which case we can conclude that quantum gravitational effects are more important than the back reaction in deciding the evolution of the star.

Again we see that the semiclassical evolution is very different from the black hole case, since the back reaction plays a crucial role during the semiclassical history of the black hole.

3   Discussion and Unresolved Issues

We see that when a spherical dust star collapses to form a naked singularity, the character of the semiclassical emission (resulting from the quantization of a massless scalar field on the dust background) is very different from the Hawking emission from a dust star collapsing to form a black hole. The duration of the semiclassical phase for the naked case is of the order of the collapse time $\sim 1/\sqrt{G\rho}$, in the comoving frame as well as for the asymptotic observer. (Since there is no event horizon preceding the formation of the singularity, there is no significant time-dilation caused by the gravitational redshift on the boundary). This time scale is of course extremely short, compared to the astronomical time scale for the evaporation of an astrophysical black hole.

With hindsight, it is not difficult to see why the semiclassical phase of a naked collapse is so different from that of collapse leading to a black hole. The central naked singularity forms extremely quickly, as seen by a far away observer, and this naked central spacetime region can only be dealt with using quantum gravity. In contrast, the asymptotic observer never gets to see the singularity inside the black hole, except during the final stages of black hole evaporation. Hence the black hole semiclassical epoch is extremely long.

Starting at about one Planck time before singularity formation, we must analyse the further evolution of the naked star using the laws of the (yet unknown) quantum theory of gravity. We thus have a novel situation where a spherical dust star (say of one solar mass) enters the quantum gravity phase with essentially all its mass intact, and unlike the black hole, it does so very much within the lifetime of the Universe. This is perhaps the first time that physicists have explicitly encountered a dynamical system whose further evolution, within the age of the Universe, cannot be understood without a knowledge of quantum gravity! For an alternative discussion of Cosmic Censorship and quantum gravity see [78]. Naked singularities in a dilaton gravity context are discussed in [79, 80].

What happens to this spherical dust star next? How does the apparently 'quantum gravitational' region near the center interact with the apparently classical regions which are away from the center? Does the entire star evaporate explosively, as suggested by the semiclassical theory, or does it settle into a black hole and then emit slowly by the standard Hawking radiation? Is there a way in the quantum theory to separate the evolution of the central region from that of the outer region? These are fascinating and challenging conceptual questions to which there are no obvious answers, and which perhaps cannot be addressed without quantum gravity. Nonetheless, they provide a valuable testbed for examining candidate theories of quantum gravity.

Motivated by such issues, Vaz et al. [81] have begun an investigation of quantum gravitational effects in spherical dust collapse leading to a naked singularity. As a midisuperspace quantum gravity model, one sets up a Wheeler-deWitt equation for the collapse, in terms of physically well-motivated canonical variables. This approach is inspired by Kuchar's canonical formalism for the Schwarzschild black hole [82]. By investigating the solutions of the Wheeler-deWitt equation for this system one hopes to address the questions raised in the previous paragraph.

As an aside, one could marvel at the richness of the dust collapse system, whose studies were first initiated by Oppenheimer and Snyder, way back in 1939. Their work on homogeneous collapse was over the years generalised to include shell-crossings and inhomogeneities, and then used as a background model in quantum field theory in curved space, and now as a quantum gravity model! We can be sure that we have not yet learnt all that dust collapse has to teach us.

Does this picture of semiclassical naked collapse persist for spherical naked singularities that form with other kinds of matter, say fluids? We think it does, because the semiclassical properties are related to the occurrence of the naked singularity in the small central region - this is a property generic to the formation of a naked singularity in spherical collapse. It is not restricted to the choice of dust as matter. Hence we believe that the choice of dust as such is irrelevant to the importance of quantum gravity in spherical naked collapse.

The other class of censorship violating spherical models which we mentioned at the beginning of this article are those which form small mass black holes, and for which the remaining mass escapes to infinity. What kind of results does one expect for the semiclassical quantization of such systems? We would like to conjecture that the semiclassical phase will be very similar to the dust naked singularity system. The small mass black hole represents a region of extremely high (Planck scale) curvature which cannot be dealt with semiclassically, and must be analysed using quantum gravity. Furthermore, this region (unlike for an evaporating black hole) is realised on a very short gravitational collapse time scale. The only physical difference from the dust case is the dispersion of the outer region, which does not constitute a significant difference in so far as the semiclassical quantisation is concerned.

Hence we can say with confidence that the picture of semiclassical spherical collapse which we have seen in the dust model is representative of censorship violating spherical collapse. The semiclassical phase of such collapse emits an insignificant amount of energy and the full evolution can only be understood in a quantum gravity framework.

Throughout the article, we have carefully emphasized that these semiclassical results apply only to naked singularities forming in spherical collapse. The picture could well be very different if naked singularities were to form in non-spherical collapse, for instance in the collapse to a elongated cigar shaped naked singularity. Under such a situation an entire region (and not just a limited central region) develops visible high curvature, and the particle creation and energy release during the process could be enormous and explosive. This is an important outstanding problem - is the semiclassical phase of highly non-spherical naked collapse similar to or very different from the semiclassical phase of naked spherical collapse? Also, the nature of gravity wave emission during the formation of aspherical naked singularities [83, 84] is a promising research problem. Of course, it should be added that our knowledge about formation of non-spherical naked singularities is at present much more restricted compared to spherical ones [85, 86, 87, 88, 89].

Finally, one could well ask whether its worth anyone's effort to investigate the possible occurrence and astrophysical properties of naked singularities. An instructive answer can be found in the history of black hole physics. Many decades ago, when a handful of relativists were trying to understand the physical properties of black holes, very few astronomers and astrophysicists took them seriously - and their efforts were often regarded as exercises in mathematical physics, irrelevant to the real world of astronomical observations. Needless to say, all this has changed with the detection of black holes in the centers of galaxies and in x-ray binaries. But its fair to add that an understanding of these detections would still be long in coming had relativists not prepared the ground by working out what black holes would look like, if they were to exist.

With regard to research on naked singularities and naked stars, we are where black hole physics was, say seventy years ago. Only further research can tell whether naked stars occur in nature. Considering the potential role of such systems as experimental testbeds for quantum gravity it seems worth the effort to explore their consequences. The possibility that gamma-ray bursts and/or some yet undiscovered astronomical object can be identified with naked stars cannot be ruled out at present [90, 91]. Even if the results of such searches turn out to be negative, one will at least have found a route to proving Cosmic Censorship.

A note on the references: No claim is made that the references are exhaustive. If there are papers references to which have been inadvertently overlooked, I would like to apologise for the same.

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